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Hyperfine structure investigation of Po^{205} and Po^{207}

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With 6 figures in the text

ABSTRACT

The atomic-beam magnetic resonance method has been used to study the magnetic and electric hfs interactions in the atomic ground state of Po^{205} and Po^{207}. For Po^{205} the conventional technique has been used and the magnetic dipole and electric quadrupole interaction constants have been obtained.

For Po^{207} a double resonance method has been introduced which has enabled the observation of an hfs separation which cannot be measured with the ordinary "flop-in" technique. In this way we have been able to compute, besides the magnetic dipole and the electric quadrupole interaction constants, also the magnetic octupole interaction constant as well as the signs of these constants. The second order perturbation energy of the non-central interaction Hamiltonian has been taken into consideration.

From the interaction constants the nuclear moments have been calculated, which are then compared with those obtained from the single-particle model.

I. Introduction

The present work represents an extension of the measurements of the nuclear spins of some polonium isotopes, recently performed at Uppsala [1], to the study of the hyperfine structure of Po^{205} and Po^{207}. Since both of these isotopes have the spin value 5/2, and since the atomic ground-state J-value of polonium is 2, there are magnetic dipole, electric quadrupole, and magnetic octupole interactions. The corresponding interaction constants may be calculated if at least three of the four hfs separations are measured.

Atomic-beam magnetic resonance investigations of radioactive atoms are most frequently accomplished with the so-called "flop-in" method. This means that only those atoms which have undergone transitions are detected and therefore, as the signal-to-background ratio is higher than in a "flop-out" arrangement, comparatively few atoms are used in an experiment. However, this method only enables the measurement of two hfs separations in the polonium isotopes studied, and therefore we have introduced a double resonance technique which has made it possible to measure another hfs separation and also to establish the signs of the interaction constants. As a double resonance experiment is quite time-consuming, it is difficult to perform with short-lived isotopes, and has only been used with Po^{207}. For Po^{205} less exact values of the magnetic dipole and electric quadrupole interaction constants have been obtained from a study of the $\Delta F = 0$ transitions.

Optical measurements have given the degree of intermediate coupling in polonium, and on the basis of these data we have been able to calculate the nuclear moments from the interaction constants. The moments have been compared with theoretically estimated values.

II. Theory

A. *Hyperfine structure*

Schwartz [2] has given a systematic review of the hyperfine structure interaction in atoms. The non-central interaction Hamiltonian is expanded in the form of tensor products:

$$\mathcal{H}_1 = \sum_k \mathbf{T}_e^{(k)} \cdot \mathbf{T}_n^{(k)}. \tag{1}$$

$\mathbf{T}_e^{(k)}$ and $\mathbf{T}_n^{(k)}$ are tensors of rank k operating in the space of electronic and nuclear coordinates, respectively. The rank corresponds to the order of multipole interaction. For the calculations we need the matrix elements (diagonal in F and m):

$$\begin{aligned}(\gamma I J F m|\mathcal{H}_1|\gamma' I' J' F m) &= \sum_k (\gamma I J F m|\mathbf{T}_e^{(k)} \cdot \mathbf{T}_n^{(k)}|\gamma' I' J' F m)\\ &= \sum_k (-1)^{I+J'+F} \begin{Bmatrix} I' & I & k \\ J & J' & F \end{Bmatrix} \sum_{\gamma''} \langle \gamma J \| \mathbf{T}_e^{(k)} \| \gamma'' J' \rangle \langle \gamma'' I \| \mathbf{T}_n^{(k)} \| \gamma' I' \rangle, \end{aligned} \tag{2}$$

where γ represents other quantum numbers necessary to specify the state. Excited nuclear states are not taken into account and therefore $\gamma'' = \gamma'$, $I = I'$.

The diagonal element of $\mathcal{H}_1$ represents the first-order energy, which is usually written:

$$W_F = \sum_k A_K M(I J; F; k), \tag{3}$$

with the normalization

$$M(I J; I + J; k) = 1. \tag{4}$$

Defining

$$\langle \mathbf{T}^{(k)} \rangle_{JJ} = \frac{(2J)!}{\sqrt{(2J-k)!\,(2J+k+1)!}} \langle J \| \mathbf{T}^{(k)} \| J \rangle, \tag{5}$$

A_k becomes

$$A_k = \langle \mathbf{T}_e^{(k)} \rangle_{JJ} \langle \mathbf{T}_n^{(k)} \rangle_{II}. \tag{6}$$

The interaction constants A_k (independent of F) are related to the frequently used constants a, b, and c:

$$A_1 = I J a, \quad A_2 = b/4, \quad A_3 = c. \tag{7}$$

The M's corresponding to $k = 1, 2, 3$ are:

$$\left.\begin{aligned} M(IJ;F;1) &= \frac{1}{2IJ}K, \\ M(IJ;F;2) &= \frac{6}{2I(2I-1)2J(2J-1)}\left[K(K+1)-\tfrac{4}{3}I(I+1)J(J+1)\right], \\ M(IJ;F;3) &= \frac{20}{2I(2I-1)(2I-2)2J(2J-1)(2J-2)} \\ \times\bigl[K^3+4K^2+\tfrac{4}{5}K\{-3I(I+1)J(J+1)&+I(I+1)+J(J+1)+3\} \\ &-4I(I+1)J(J+1)\bigr], \end{aligned}\right\} \quad (8)$$

where
$$K = F(F+1) - I(I+1) - J(J+1). \quad (9)$$

Since eq. (2) gives non-vanishing matrix elements between different electronic states, there exist second and higher order perturbations of $\mathcal{H}_1$.

If ΔE is the energy difference between an exited state and the ground state (used in our experiments), we get the second order energy,

$$W_F^{(2)} = \frac{-1}{\Delta E}\Bigl[\sum_k (\gamma I J F \,|\, \mathbf{T}_e^{(k)} \cdot \mathbf{T}_n^{(k)} \,|\, \gamma' I J' F)\Bigr]^2, \quad (10)$$

where m is omitted since the matrix elements are independent of m. As the octupole interaction is several orders of magnitude smaller than the dipole and quadrupole interactions, the summation is made for $k=1$ and $k=2$. The small values of $W_F^{(2)}$ obtained justify a non-relativistic treatment which is easily done in the LS-coupling scheme. Trees [3] has given the operators:

$$\mathbf{T}_e^{(1)} = \sum_i \bigl[\mathbf{l}_i^{(1)} - \sqrt{10}\,(\mathbf{s}^{(1)}\mathbf{C}^{(2)})_i^{(1)}\bigr], \quad (11)$$

$$\mathbf{T}_e^{(2)} = \sum_i (r_i^{-3})\, 2\, (\mathbf{C}^{(2)})_i. \quad (12)$$

All that we have to calculate is the reduced matrix elements:

$$\langle \gamma J \| \mathbf{T}_e^{(k)} \| \gamma' J' \rangle, \quad (13)$$

and the six-j symbols in eq. (2). By means of the measured values of A_k and eqs. (5, 6) the reduced matrix elements of $\mathbf{T}_n^{(k)}$ are eliminated, and we can get the non-diagonal elements of $\mathcal{H}_1$ needed in eq. (10). The formulas needed for the calculation of (13) are found in standard textbooks [4, 5]. As the electronic configuration of polonium is p^4, the p^2 configuration wave functions are used in the calculations, and the sign of the electric quadrupole operator is thereby changed. Since there is a high degree of intermediate coupling, the electronic wave functions are written as linear combinations,

$$\psi_i = b_{1i}\varphi_{1i} + b_{2i}\varphi_{2i}, \quad (14)$$

and the matrix elements (13) become:

$$\langle \psi_i \| \mathbf{T}_e^{(k)} \| \psi_j \rangle = \sum_{l,t} b_{li} b_{tj} \langle \varphi_{li} \| \mathbf{T}_e^{(k)} \| \varphi_{tj} \rangle. \quad (15)$$

B. *Wave functions in intermediate coupling*

As known from optical measurements [6, 7] the coupling between the outer electrons in polonium is intermediate between the LS- and jj-extremes. The wave functions can be expressed as linear combinations of the eigenfunctions in either of the extreme couplings. The coefficients are obtained from the measured values of the spin-orbit (ζ) and electrostatic interaction (F_2) constants. The measured g_J-value, 1.39609 [1], could also be used for these calculations, but since the evaluation of the g_J corrections is difficult, the first method is preferred.

The electronic ground-state configuration of polonium is, apart from possible configuration interaction, $6s^2 6p^4$. This gives rise to five possible states, denoted by ${}^3P_{2,1,0}$, ${}^1D_2$, and ${}^1S_0$ in LS-coupling. As J is assumed to be a good quantum number, we can express the wave functions in intermediate coupling (here denoted by $'$) as:

$$\left.\begin{aligned}
{}^1D_2' &= b_{11}\,{}^1D_2 + b_{21}\,{}^3P_2,\\
{}^3P_2' &= b_{12}\,{}^1D_2 + b_{22}\,{}^3P_2,\\
{}^3P_1' &= {}^3P_1,\\
{}^1S_0' &= b_{11}'\,{}^1S_0 + b_{21}'\,{}^3P_0,\\
{}^3P_0' &= b_{12}'\,{}^1S_0 + b_{22}'\,{}^3P_0,
\end{aligned}\right\} \qquad (16)$$

where ${}^1D_2$ etc. means the corresponding wave function.

Condon and Shortley [8] have given the energy matrices in the LS-coupling scheme (with ${}^3P_1$ as reference):

$$\left.\begin{aligned}
H_{J=2} &= \begin{pmatrix} 6F_2 - \frac{1}{2}\zeta & -\frac{\sqrt{2}}{2}\zeta \\ -\frac{\sqrt{2}}{2}\zeta & -\zeta \end{pmatrix},\\
H_{J=0} &= \begin{pmatrix} 15F_2 - \frac{\zeta}{2} & \sqrt{2}\,\zeta \\ \sqrt{2}\,\zeta & \frac{\zeta}{2} \end{pmatrix}.
\end{aligned}\right\} \qquad (17)$$

The energy eigenvalues (λ_k) are:

$$\left.\begin{aligned}
\left.\begin{matrix} {}^1D_2' \\ {}^3P_2' \end{matrix}\right\} &= 3F_2 - \tfrac{3}{4}\zeta \pm \sqrt{9F_2^2 + \tfrac{3}{2}F_2\zeta + \tfrac{9}{16}\zeta^2},\\
\left.\begin{matrix} {}^1S_0' \\ {}^3P_0' \end{matrix}\right\} &= \tfrac{15}{2}F_2 \pm \sqrt{\tfrac{225}{4}F_2^2 - \tfrac{15}{2}F_2\zeta + \tfrac{9}{4}\zeta^2},
\end{aligned}\right\} \qquad (18)$$

and the coefficients b_{ik} are:

$$b_{1k} = \frac{\lambda_k - a_{22}}{\sqrt{(\lambda_k - a_{22})^2 + a_{21}{}^2}}, \quad b_{2k} = \frac{a_{21}}{\sqrt{(\lambda_k - a_{22})^2 + a_{21}{}^2}}. \qquad (19)$$

Here a_{ij} are the matrix elements of (17).

The wave functions can also be expressed as linear combinations of the jj-coupling eigenfunctions. The polonium ground state ${}^3P_2'$ is written:

$$ {}^3P_2' = c_1 \left(\tfrac{3}{2}\tfrac{3}{2}\right)_2 + c_2 \left(\tfrac{3}{2}\tfrac{1}{2}\right)_2. \tag{20}$$

The transformation matrix from LS- to jj-coupling is:

$$\alpha = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}, \tag{21}$$

which gives c_1 and c_2 when the b_{ik}'s are known.

With $\zeta = 12379$ K and $F_2 = 1096$ K [7] we get:

$$\begin{aligned} &b_{11} = 0.8914, \quad b_{21} = -0.4532, \quad b'_{11} = 0.7467, \quad b'_{21} = 0.6651, \\ &b_{12} = -0.4532, \quad b_{22} = -0.8914, \quad b'_{12} = -0.6651, \quad b'_{22} = 0.7467, \\ &\qquad c_1 = -0.9895, \quad c_2 = -0.1446. \end{aligned}$$

C. *Calculation of the nuclear moments*

The nuclear moments are calculated from the measured values of the interaction constants. Breit and Wills have derived the following formula for the magnetic dipole interaction constant [9]:

$$A_1 = I\,J\,[c_1^2 a' + \tfrac{1}{4} c_2^2 (3a' + a'') + \sqrt{2}\, c_1 c_2 a'''], \tag{22}$$

where

$$\left.\begin{aligned} a' &= -\frac{16}{15}\frac{\mu_0}{4\pi} g_I \mu_B^2 F' \langle r^{-3}\rangle_{cl}, \\ a'' &= -\frac{16}{3}\frac{\mu_0}{4\pi} g_I \mu_B^2 F'' \langle r^{-3}\rangle_{cl}, \\ a''' &= \frac{1}{3}\frac{\mu_0}{4\pi} g_I \mu_B^2 G \langle r^{-3}\rangle_{cl}, \end{aligned}\right\} \tag{23}$$

μ_B is the Bohr magneton, $g_I = -\mu_I/I\,\mu_B$, and F', F'', and G relativistic corrections tabulated by Kopfermann [10].

The value $\langle r^{-3}\rangle_{cl}$ can be obtained from the spin-orbit interaction constant [11]:

$$\langle r^{-3}\rangle_{cl} = \frac{4\pi\zeta}{2\mu_0 H_r Z_i \mu_B^2}. \tag{24}$$

H_r is a relativistic correction [10] and Z_i is the "effective" nuclear charge. For p-electrons, Barnes and Smith [12] have given the value $Z_i = Z - n = 78$ for polonium.

The electric quadrupole moment, Q, is related to the quadrupole interaction constant by

$$A_2 = -\frac{e^2}{4\cdot 4\pi\varepsilon_0} Q \left\langle \sum_i \frac{3\cos^2\theta_i - 1}{r_i^3} \right\rangle_{JJ}, \tag{25}$$

where the summation is performed over all electrons and the average is taken in the state $m_J = J$. Following the method described by Schüler and Schmidt [11] we obtain:

$$\left\langle \sum_i \frac{3\cos^2\theta_i - 1}{r_i^3} \right\rangle_{JJ} = \frac{2}{5}\left(c_2^2 R' - c_1 c_2 \cdot 2\sqrt{2}\, S\right) \langle r^{-3} \rangle_{cl}, \tag{26}$$

where the relativistic correction factors R' and S are tabulated in reference [10].

Since the magnetic octupole interaction constant for a single $p_{3/2}$ electron, $A_3(p_{3/2})$, has been calculated by Schwartz [2], we have derived an expression for the relation between A_3 and $A_3(p_{3/2})$:

$$A_3 = (-2c_1^2 + c_2^2)\, A_3(p_{3/2}), \tag{27}$$

$$A_3(p_{3/2}) = \frac{\mu_0 \cdot 16}{4\pi \cdot 3 \cdot 5 \cdot 7}\, \Omega\, \mu_B\, \frac{Z_i^2}{a_0^2}\, T \langle r^{-3} \rangle_{cl}, \tag{28}$$

where Ω is the nuclear magnetic octupole moment, and T a relativistic correction given by Casimir [13].

Eq. (27) was obtained from (5) and (6) by expanding $\langle J \| \mathbf{T}_e^{(3)} \| J \rangle$ in $\langle j \| \mathbf{T}_e^{(3)} \| j \rangle$, which is easily done without knowledge of the explicit expression for $\mathbf{T}_e^{(3)}$ [5].

III. Experimental Procedure

A. *General*

The six-pole focusing atomic-beam apparatus used in these experiments has been described earlier [14] and is the same as was used for the spin measurements [1]. The production and detection of an atomic beam of polonium was also described in reference [1].

The effect of an external magnetic field on the electronic and nuclear moments is expressed by the Hamiltonian:

$$\mathcal{H}_2 = g_J \mu_B \mathbf{J} \cdot \mathbf{B} + g_I \mu_B \mathbf{I} \cdot \mathbf{B}, \tag{29}$$

where $g_J = -\mu_J / J\,\mu_B$. The energy levels can be obtained from diagonalization of the total Hamiltonian $\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2$. To surmount the complex computational problems, a routine has been set up for the IBM 650 computer. This diagonalizes the submatrix for a given m-value containing the elements of the magnetic dipole and electric quadrupole interactions, and the first term in eq. (29). The magnetic octupole interaction, the g_I-term, and the non-diagonal elements of $\mathcal{H}_1$ are then considered as corrections to the computed energies.

A schematic Breit-Rabi diagram for Po^{207} ($I = \frac{5}{2}$, $J = 2$) is given in Fig. 1. It is also valid for Po^{205} if the sign of the magnetic dipole interaction constant A_1 is positive as for Po^{207}. In the atomic beam apparatus used, only atoms which have a negative "effective" magnetic moment in the A-magnet and a positive "effective" magnetic moment in the B-magnet reach the detector. With the oscillatory radio frequency field perpendicular to the static magnetic field the selection rules are $\Delta F = 0, \pm 1, \Delta m = \pm 1$. From this we may conclude that we can observe the following transitions:

$$(F, m) = (9/2,\ -3/2) \leftrightarrow (7/2,\ -5/2) \text{ and } (7/2,\ -1/2) \leftrightarrow (5/2,\ -3/2).$$

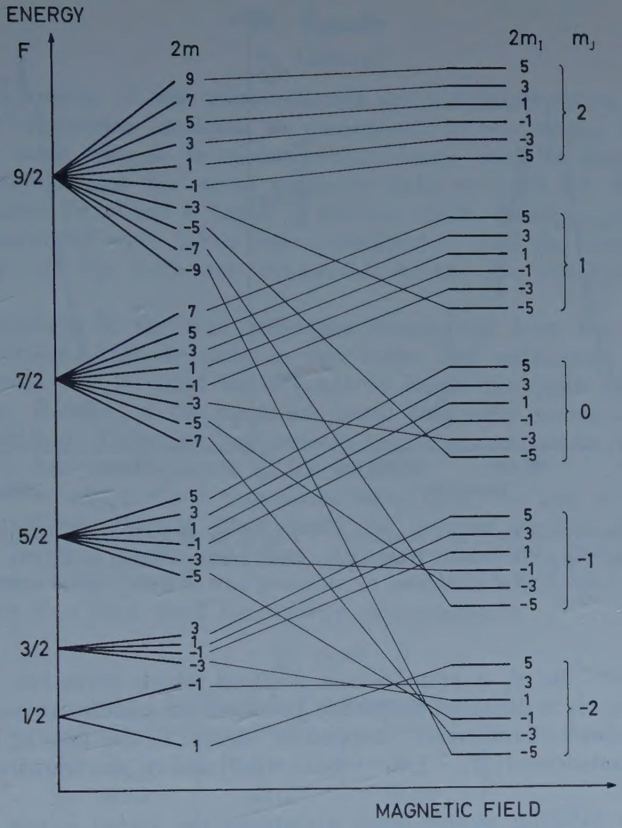

Fig. 1. Schematic Breit-Rabi-diagram for the system $I=5/2$, $J=2$ of Po^{207} ($A_1>0$).

However, if an excessive amount of radio frequency power is used, multiple quantum transitions ($\Delta m \geqslant 2$) can be induced [1]. This means that we can observe transitions for which $\Delta F=0$ ($F=9/2$ and $F=7/2$).

A study of the above mentioned transitions will give the magnetic dipole and electric quadrupole interaction constants. The octupole interaction is, however, several orders of magnitude smaller and cannot be determined, unless another transition, for which $\Delta F=1$, can be observed. Therefore a double resonance technique has been introduced.

B. *Double resonance experiments*

In a conventional "flop-in" type atomic-beam apparatus with dipole A- and B-magnets only those transitions are observed which merely change the sign of the effective magnetic moment, i.e. $(\mu_{\text{eff}})_A = -(\mu_{\text{eff}})_B$, ($\mu_{\text{eff}}$ denotes the strong field value). If there are two oscillatory fields (here denoted rf_1 and rf_2) instead of one, a double resonance experiment may be performed. King and Jaccarino [15] have used what they call a "flop-out on flop-in" method to measure the hfs separations

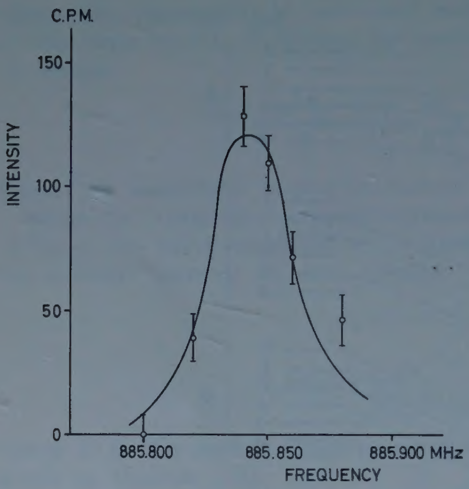

Fig. 2. Resonance curve for Po^{207} in a magnetic field corresponding to $\mu_B B/h = 2.095$ MHz. $(F, m) = (9/2, -3/2) \leftrightarrow (7/2, -5/2)$.

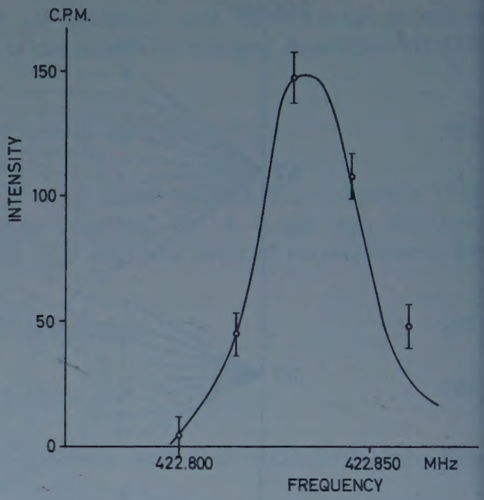

Fig. 3. Resonance curve for Po^{207} in a magnetic field corresponding to $\mu_B B/h = 2.100$ MHz. $(F, m) = (7/2, -1/2) \leftrightarrow (5/2, -3/2)$.

for Br^{79} and Br^{81}. In rf_1 a transition is induced which gives rise to a signal at the detector. In rf_2 a different frequency produces an additional transition, which reduces the amount of refocused "flopped-in" beam. In the case of Po^{207} rf_1 could e.g. induce the transition $(7/2, -1/2) \leftrightarrow (5/2, -3/2)$ and rf_2 the transition $(5/2, -3/2) \leftrightarrow (5/2, -5/2)$.

In a six-pole focusing atomic-beam apparatus the signal is not very sensitive to the magnitude of μ_{eff} in the B-magnet, and therefore a somewhat different double resonance method is used. The field rf_1 induces a transition on atoms in a state with $\mu_{\text{eff}} < 0$ to another state with $\mu_{\text{eff}} = 0$. In rf_2 some of the atoms undergo further transition to a state with $\mu_{\text{eff}} > 0$, and a signal is obtained at the detector.

Since the order in which rf_1 and rf_2 are applied is essential to obtain a signal, a double resonance experiment can be used to determine the sign of A_1. If A_1 were negative, the energy level diagram would be reversed, and rf_1 and rf_2 would have to be interchanged. Therefore, when we induce the transition $(7/2, 1/2) \leftrightarrow (5/2, -1/2)$ in rf_1 and $(5/2, -1/2) \leftrightarrow (5/2, -3/2)$ in rf_2, a signal at the detector implies a positive value of A_1. A further check of the sign of the interaction constants is obtained from a "flop-out on flop-in" experiment. In rf_1 the transition $(7/2, -1/2) \leftrightarrow (5/2, -3/2)$ is induced. The obtained signal decreases, when the transition $(5/2, -3/2) \leftrightarrow (5/2, -1/2)$ is induced in rf_2, if rf_1 and rf_2 are applied in the right order.

Since the sign of A_1 has been established, investigations to determine the octupole interaction constant can be performed. For Po^{207} this means a measurement of the hfs separation between $F = 5/2$ and $F = 3/2$. For that reason we induce the transitions $(7/2, 1/2) \leftrightarrow (5/2, -1/2)$ and $(5/2, -1/2) \leftrightarrow (3/2, +1/2)$ in rf_1 and rf_2, respectively.

IV. Results

A. *General*

The primary results of our measurements are tabulated below. The errors of the measured frequencies (enclosed in parenthesis in the tables) have been estimated in the same way as in earlier atomic beam work at this institute [16]. For the calibration of the static magnetic field we used K^{39}. The uncertainty was usually taken as about a tenth of the line-width. Small magnetic field drifts during an experiment were taken into consideration and the error was increased correspondingly. If the drift was comparable to the line-width, the experiment was repeated.

The determination of the true resonance frequencies from the polonium lines was more uncertain. This depends on two facts. The radioactive detection gives statistical errors, and it could be difficult to locate the peak of the resonance curves exactly. Besides, if the optimum radio frequency power is exceeded the line may be shifted. Therefore, we usually take the uncertainty of the frequency of a symmetric line as a quarter of its width.

Since for Po^{205} only $\Delta F = 0$ transitions were used, we had to examine which levels were involved. Several lines could be induced in low magnetic fields, corresponding to two-, three-, and four-quantum transitions. From the frequency relations between these lines it was possible to conclude which was a two-quantum transition. This was then used for further measurements.

B. *Po^{207}*

Table of Δ $F = 0$ transitions.

Transition F, m, m'	ν_K MHz	$\mu_B B/h$ MHz	ν_{Po} MHz	ν_{calc} MHz	
				$\mu_I = 0$	$\mu_I = 0.5$ nm
9/2, −3/2, −7/2	29.581 (5)	50.061 (8)	31.909 (10)	31.915 (5)	31.912 (5)
	60.668 (5)	89.890 (7)	58.526 (10)	58.539 (5)	58.535 (5)
	99.730 (9)	129.981 (10)	86.455 (25)	86.472 (9)	86.465 (9)
	149.138 (5)	171.983 (5)	116.988 (15)	117.014 (4)	117.006 (4)
7/2, −1/2, −5/2	29.560 (5)	50.030 (8)	31.510 (7)	31.512 (5)	31.510 (5)
	60.757 (5)	89.992 (7)	60.920 (12)	60.908 (5)	60.906 (5)
	99.730 (9)	129.981 (10)	95.270 (25)	95.277 (9)	95.272 (9)

ν_K is the K^{39} resonance frequency and ν_{calc} is the Po resonance frequency, calculated from ν_K and the measured interaction constants (see below). A comparison between ν_{calc} and ν_{Po} shows that a positive nuclear magnetic moment is more probable than a negative one, in agreement with the determined sign of A_1.

From the table below we get $\Delta\nu$, the hfs separations. As the non-relativistic second order energy calculated from eq. (10) is small (< 3 kHz) compared to the experimental uncertainties, the relativistic effects are taken into account as an increase of the uncertainties with 2 kHz and the corrected hfs separations become:

$$\Delta\nu_{9/2} = 884.785\ (13)\ \text{MHz},$$
$$\Delta\nu_{7/2} = 421.950\ (8)\ \text{MHz},$$
$$\Delta\nu_{5/2} = 158.567\ (12)\ \text{MHz}.$$

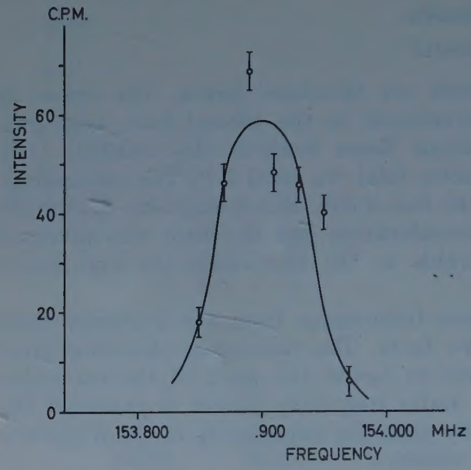

Fig. 4. Resonance curve for Po^{207} in a magnetic field corresponding to $\mu_B B/h = 8.049$ MHz obtained in a double resonance experiment. $(F, m) = (5/2, -1/2) \leftrightarrow (3/2, 1/2)$.

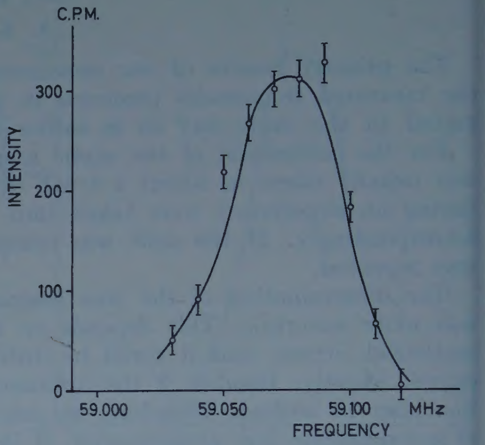

Fig. 5. Double quantum transition for Po^{205} in a magnetic field corresponding to $\mu_B B/h = 90.005$ MHz. $(F, m) = (9/2, -3/2) \leftrightarrow (9/2, -7/2)$.

Table of $\Delta F = 1$ *transitions.*

Transition F, m, F', m'	ν_K MHz	$\mu_B B/h$ MHz	ν_{Po} MHz	$\Delta\nu$ MHz
$9/2, -3/2, 7/2, -5/2$	0.521 (8) 1.056 (4)	1.039 (16) 2.095 (8)	885.334 (15) 885.842 (13)	884.804 (17) 884.776 (14)
$7/2, -1/2, 5/2, -3/2$	0.364 (5) 1.009 (5) 1.059 (5)	0.726 (10) 2.003 (10) 2.100 (10)	422.260 (10) 422.820 (25) 422.834 (5)	421.951 (12) 421.977 (26) 421.949 (7)
$5/2, -1/2, 3/2, 1/2$	1.533 (7) 2.090 (5) 4.136 (7)	3.034 (14) 4.121 (10) 8.049 (14)	157.160 (15) 156.520 (15) 153.900 (20)	158.559 (17) 158.563 (16) 158.580 (22)

The interaction constants A_k are obtained from eqs. (3, 8).

$$A_1 = 697.754\,(10)\ \text{MHz},$$
$$A_2 = 95.137\,(4)\ \text{MHz},$$
$$A_3 = -0.0120\,(10)\ \text{MHz}.$$

The nuclear moments are calculated from eqs. (22–27)

$$(\mu_I = 0.27\ \text{nm}),$$
$$Q = 0.28 \cdot 10^{-24}\ \text{cm}^2,$$
$$\Omega = 0.11 \cdot 10^{-24}\ \text{nm} \cdot \text{cm}^2.$$

The nuclear magnetic dipole moment is very uncertain, mainly because of possible configuration interaction. For Bi^{209} optical measurements [17] have enabled

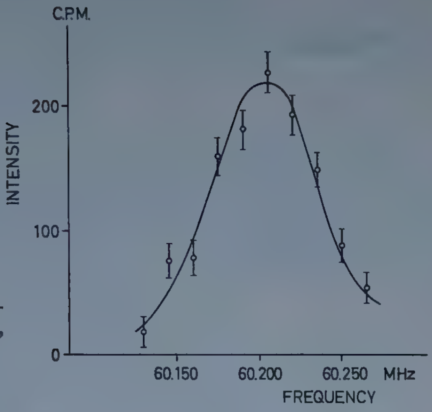

Fig. 6. Double quantum transition for Po^{205} in a magnetic field corresponding to $\mu_B B/h = 89.960$ MHz. $(F, m) = (7/2, -1/2) \leftrightarrow (7/2, -5/2)$.

the determination of a' and a'' which do not at all agree with the values from eq. (23). From these values and (22, 23) the magnetic moment of Po^{207} has been calculated with the result $\mu_I = 1.3$ nm. This gives an estimate of the uncertainty of the μ_I-value. Marrus *et al.* [18] made an estimate of the configuration interaction in Am^{241}. However, as they pointed out, lack of knowledge of the parameters involved, made the calculations very uncertain, and, in fact, the correction only enhanced the discrepancy. Therefore, we have made no efforts to estimate the effect of configuration interaction in polonium.

The most uncertain parameter in the determination of the electric quadrupole and magnetic octupole moments is Z_i. From [12] we conclude that the uncertainty of Z_i is about 10 per cent. The Sternheimer correction [19], which is not known, has not been taken into account. The uncertainty in Ω, resulting from possible configuration interaction which may effect $W_F^{(2)}$, is probably less than 10 per cent, and has not been taken into account.

C. *Po*^{205}

Table of transitions.

Transition F, m, m'	ν_K MHz	$\mu_B B/h$ MHz	ν_{Po} MHz
$9/2, -3/2, -7/2$	17.760 (9) 29.540 (13) 60.768 (19)	31.936 (15) 50.002 (20) 90.005 (24)	20.213 (10) 32.010 (10) 59.075 (15)
$7/2, -1/2, -5/2$	17.768 (9) 29.539 (13) 60.729 (24)	31.950 (15) 50.000 (20) 89.960 (30)	19.425 (10) 31.330 (10) 60.205 (20)

From the above table we calculate:

$$A_1 = 673.2\ (2.2)\ \text{MHz},$$
$$A_2 = 57.3\ (1.2)\ \text{MHz},$$
$$(\mu_I = 0.26\ \text{nm}),$$
$$Q = 0.17 \cdot 10^{-24}\ \text{cm}^2.$$

Because of the uncertainty of the nuclear dipole moment, A_1 and A_2 were calculated under the assumption that $\mu_I = 0$.

V. Discussion

The numbers of protons and neutrons in the studied polonium isotopes are near the magic numbers 82 and 126. Therefore the nuclear shell model is probably a good approximation. The spin-values [1] are consistent with the following ground-state configurations:

$$\mathrm{Po}^{207}\colon (\pi h_{9/2})^2 (\nu p_{1/2})^{-2} (\nu f_{5/2})^{-1},$$
$$\mathrm{Po}^{205}\colon (\pi h_{9/2})^2 (\nu p_{1/2})^{-2} (\nu f_{5/2})^{-3}.$$

The calculated values of the magnetic dipole moments are so uncertain that a comparison with theory is useless. It was hoped that a study of the $\Delta F = 0$ transitions in Po^{207} would give the dipole moment, but unfortunately the precision of our measurements was not high enough. Efforts were made to observe resonances in stronger magnetic fields, but the available radio frequency power was too low. From our data it is clear, however, that the necessary precision is within reach mainly because of the fact that all interactions except that of the nuclear magnetic dipole moment with an external magnetic field, are well known.

In a review article by Blin-Stoyle the nuclear moments associated with different nuclear models are discussed [20]. The extreme single-particle value of the electric quadrupole moments of Po^{205} and Po^{207} is about $-0.0004 \cdot 10^{\ 24}$ cm^2, several orders of magnitude lower than the observed values. These can, however, be explained on basis of the individual particle model. An odd-neutron nucleus with protons outside closed shells is expected to have an appreciable electric quadrupole moment. On the other hand a half-filled shell like $(\nu f_{5/2})^{-3}$ in Po^{205} should have no quadrupole moment. This may explain the difference between the quadrupole moments of Po^{205} and Po^{207}.

The measured octupole moment of Po^{207} is compared with the extreme single-particle value. This is $0.20 \cdot 10^{-24}$ nm · cm^2, about twice the measured value. However, in the same way as magnetic dipole moments deviate from the Schmidt lines, octupole moments tend to deviate from the single-particle lines.

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